Problems

1. At a primeval market one could exchange a mammoth skin for two saber-toothed tiger skins, and a peacock feather skirt for three stone spears. At another market, located one day away from the first one, one could exchange a mammoth skin for three peacock skirts, and a tiger skin for four spears. All exchanges could be done both ways. A hunter brings a mammoth skin to the first market and wants to exchange it for four tiger skins. Could he do this in 33 days?

2. Find $\lim\limits_{x\to\infty} x^{1/ln(x)}$.

3. Is the function
$$\ln(x+\sqrt{x^2+1})$$
even, odd or neither of these two?

4. Bassil executes a parachute jump with constant speed v, while his friend Michael rides a Ferris wheel which rotates with constant speed. (The initial point of Bassil's jump is higher than the uppermost point of the Ferris wheel). At the moment of Bassil's landing the cabin in which Michael sits is at the lowest point (at the ground level). It is known that during Bassil's descent the friends were at the same height precisely four times. Find the vertical component of Michael's speed at the moment when they found themselves at the same height for the first time. (You may consider the two boys to be points).

5. How many solutions does the equation $x^2 = 2^x$ have ?

6. Let A and B be arbitrary non-zero matrices of the second order. Prove that there exists a matrix C such that $ACB \neq 0$.

7. Find the minimal value of the expression
$$\sqrt{(x-1960)^2+y^2}+\sqrt{x^2+(y-441)^2}$$
.

8. Find a non-zero polynomial with integer coefficients with a root
$$\sqrt[5]{\sqrt{2}+1}-\sqrt[5]{\sqrt{2}-1}.$$

9. Two numbers are chosen randomly in the segment $[0, 1]$. Calculate the probability that the square of the first number is greater than the second one.

10. The axes of two cylinders, the radiuses of which equal 1, intersect and are perpendicular. Find the volume of the intersection of the cylinders.

11. A professor has formulated n statements A_1, A_2, ..., A_n. He gives his postgraduate students subjects for dissertations: "A_i implies A_j". No dissertation should be a direct logical consequence of previously given ones. What is the maximal number of students that the professor could have?

12. Let $f(x, y)$ be an infinitely differentiable function of two variables with a local minimum at the origin, and let this function have no other *critical* points. Is it true that the minimum is global? (A point is called critical if both partial derivatives $\partial f/\partial x$ and $\partial f/\partial y$ at it equal zero).