Olympiad, April 14, 2011

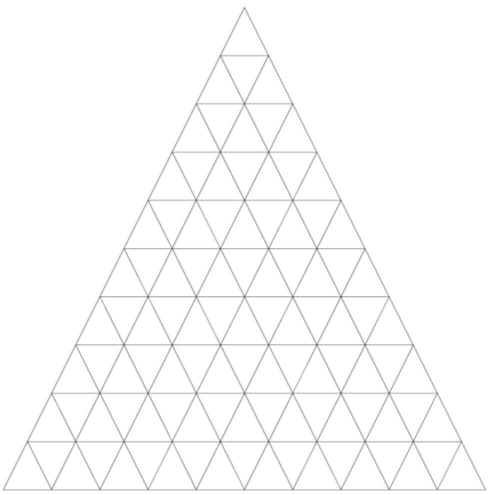
Please write your name and your registration number on the title page, draw and fill in the following table

	1	2	3	4	5	6	7	8	9	10
Answer										

Do not write anything else on the title page. Problems with empty "Answer" cells in the table will not be checked. Solutions for different problems please write down on different sheets. To avoid confusion please write the registration number on each sheet. **Good luck!**

Problem 1. Find the number of local extrema of the function $y = f\big(f(x)\big)$, if $f(x) = x^3 - 3x + 1$.

Problem 2. What maximal number of rooms one could traverse in the triangular labyrinth (see the figure) if permitted to pass from each room to each adjacent one, but forbidden to enter any room twice?



Problem 3. Let O be the center of a regular 2011- gon $A_1 \ldots A_{2011}$, and X be an arbitrary point. How many times greater is the length of the sum of vectors which connect point X with vertices of the polygon, than the length of the vector which connects point X with point O?

Problem 4. A set of points $A_n(x_n; y_n)$ $(n = 1, 2, \ldots, 2009)$ is given on the plane. Here $x_n = \left(1 - \frac{n}{2010}\right) \cdot \cos\frac{\pi n}{502}$; $y_n = \left(1 - \frac{n}{2010}\right) \cdot \sin\frac{\pi n}{502}$. Let us call a point $A_j\left(x_j; y_j\right)$ dominant over a point $A_i(x_i; y_i)$, if $x_j \geq x_i$ and $y_j \leq y_i$ for $j \neq i$. How many points are there such that no point dominates over them.

Problem 5. Find the sum of coefficients of the odd degrees of the Taylor series for the function $f(x) = \frac{2x+3}{x^2-2x+2}$.

Problem 6. In the triangle ABC lengths of sides are the following: $|AB| = l_1$, $|BC| = l_2$, $|AC| = l$. On the side AC a point M is given such that $|AM| = x$. Let us denote the angle $AMB = \theta(x)$. Find $\int_0^l \cos\theta(x)dx$, if $l_1 = 64$ and $l_2 = 81$.

Problem 7. Find the minimal value of the expression $16\frac{x^3}{y} + \frac{y^3}{x} - \sqrt{xy}$.

Problem 8. Find $\int_0^2 \min_{y\in[0;2]}(3x + |3x - y|)dx$ ($\min_{t\in[a;b]} f(t)$ denotes the minimal value of the expression $f(t)$ on the segment $[a; b]$).

Problem 9. Find the area of the convex polygon on the complex plane, the vertices of which are the roots of the equation $1 + z + z^2 + z^3 + z^4 + z^5 = 0$.

Problem 10. A sequence $\{x_n\}$ is determined by the relation: $x_n(1 - x_{n-1}) = x_{n-1}$ ($n \geq 1$). It is known that $x_{2011} = 2011$. Find x_0.