# Team Olympiad. December 13, 2011

**1.** The road from point $A$ to point $B$ goes uphill for $3-\sqrt{2}$ km, downhill for $2+\sqrt{3}$ km, and is horizontal for the rest of the way. A traveler covered the distance from $A$ to $B$ and back again in 4 hours. He walked uphill at a speed of 3 km/h, downhill at a speed of 6 km/h, and horizontally at a speed of 4 km/h. What is the length of the road from $A$ to $B$ (in kilometers)?

**2.** The regular 2011-gon of side 1 circumscribes a circle and is inscribed in a circle. Find the area of the ring between the two circles.

**3.** In a convex $n$-gon no three diagonals intersect at one point. What is the number of points of intersection of the diagonals?

**4.** Find the 300-th digit after the decimal point of the number $\sqrt[3]{0.999\ldots9}$ (a hundred nines).

**5.** Calculate the sum of the series:

$$\sum_{n=1}^{\infty} \frac{n}{(n+2)!}\,.$$

**6.** When I cross the street in a place other than the pedestrian crossing, the probability of being hit by a car is 0.01. What is the probability of being hit by a car if I cross the street in such a place 100 times?

**7.** Find all matrices $A$, such that

$$A^{2011} = \begin{pmatrix} -4021 & -4022 \\ 4022 & 4023 \end{pmatrix}$$

**8.** Evaluate the integral with error of no more than 20%.

$$\int_{-\pi/2}^{\pi/2} \sin^{100} x dx.$$